Rumor Spreading with Consistent Actors:
EE 376a Final Report

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Abstract

We investigate a model for rumor-spreading in a social network with actors that consistently output the same rumor. Modifying a previously developed model, we introduce liars and truth-tellers as "consistent actors". Even when a small portion of the network is composed of "consistent actors", they have a noticeable effect on dominant opinions. However, these consistent actors have little effect on the entropy within the memory of other nodes. From this, we conclude that the presence of consistent actors allows rumors to still spread while subtly forcing the dominant opinion to conform to their choice of rumor.

1 Introduction

Social networks influence nearly every aspect of the daily decisions faced by people, from which products to purchase to which political candidates to support. Thus, understanding how ideas spread in these networks is of great interest. Studying rumor spreading in networks models the effects of interest, as "rumors" can represent ideas, opinions, or other types of content. The underlying network models themselves are well-suited for capturing the interactions between individuals, from peer-to-peer interactions to influencer-to-peer interactions.

In social networks, rumors can be distorted through failures in communication, misremembered information, or changing opinions. However, not all actors in a network are subject to this equally. In particular, there are often "consistent actors" who only spread one rumor. In applications, these actors could be loyal customers or voters, or could be paid sponsors. This suggests that the rumor spreading model should account for certain people, at all levels of influence, consistently spreading the same rumor. In many applications, such as advertising and political elections, there may be consistent actors with opposing messages, competing to influence public opinion. This work modifies an existing rumor spreading model to study the effects of consistent actors in a network, especially with competing consistent actors.

2 Related Work

Wang *et al.* developed a model of rumor-spreading which this work extends to include consistent actors. Their work models individuals as nodes, with the underlying network structure as a Barabási-Albert scale-free network. This network structure is well-suited to modeling social networks, as it features "hub" nodes, which act as the important figures in social networks. Because of the power law governing the degree of nodes in the network, there are a variety of hubs with different levels of influence, capturing the nuance in the relative influential power of different key actors [1].

With this network structure, Wang *et al.* use a fixed memory size, probability of distortion based on entropy within a person's memory, and probability of rumor acceptance based on relative social standing to model rumor spreading effects. They demonstrate the model's ability to recreate effects seen in real social networks, such as opinion fragmentation in low-trust networks and opinion conformity in high-trust networks [2].

3 Model

This work starts with a model very similar to the one developed by Wang *et al* [2].Unless otherwise stated, the numerical parameters defined here are used in both the original work and this work. The network is generated as a BA scale-free network. While the original study uses $m_0 = 5$ initializing nodes, $m = 2$ links per new node, and $N = 3000$ total nodes, this work differs in using only $N = 300$ total nodes due to computational limitations. Each "rumor" is represented as a binary string of length $s = 5$, and so there are $2^5 = 32$ possible rumors. Each individual can remember the most recent $L = 320$ pieces of information told to them. If the frequency of the i^{th} possible rumor in an individual's memory is given by f_i, then the Shannon entropy within the memory of individual n is given by

$$H_n = -\sum_i f_i \log_2(f_i)$$

The probability of individual n distorting information is

$$P_n = \left(1 + \exp\left(\frac{H_{max} - H_n}{H_{max}}\right)\right)^{-1}$$

where $H_{max} = s$ is the maximum possible entropy and $K \geq 0$ is a tunable parameter that offers control over the probability of distortion across all nodes.

Upon receiving a piece of information from individual n, individual m will accept this piece of information into m's memory with probability

$$\eta_{mn} = \frac{k_n^\beta}{\max l \in \mathcal{N}(m) k_l^\beta}$$

where k_n is the degree of node n, $\mathcal{N}(m)$ is the set of neighbors of node m, and β is a tunable parameter that controls whether individuals trust highly connected neighbors or isolated neighbors.

With this structure in place, the rumor spreading takes place as follows. An initial rumor propagator is chosen uniformly at random, and given the initial rumor $'00000'$. On each time step, every node attempts to spread a rumor. For a fixed node n, if there are no pieces of information in the memory bank of n, it does nothing. If there is at least one piece of information in its memory bank, node n chooses the rumor with the highest frequency in its memory, with ties broken arbitrarily. With probability P_n, this rumor is mutated by a bitflip with location chosen uniformly at random. The mutated rumor replaces the first occurrence of the original rumor in the individual's memory. The selection and mutation process happens simultaneously for all nodes. Next, this rumor (mutated or not) is then spread to each neighbor of of n, and each neighbor m accepts with probability η_{mn}. If the information is accepted, then the new rumor is added to the memory of m. If m now has greater than the allowed capacity L in the memory bank, then the oldest pieces of information are deleted until the memory satisfies the limit. All memory updates occur simultaneously. Note that memory updating occurs after rumor selection, mutation, and spreading for all nodes.

The above model is that developed by Wang *et al* [2]. To this model, this work introduces "liar" nodes and "truth-teller" nodes. Truth-teller nodes spread the rumor $'00000'$ and liar nodes spread $'11111'$; otherwise, their dynamics are identical. The particular nodes set as liar and truth-teller nodes are chosen in various ways, with more details in the descriptions of the relevant experiments. The liar and truth-teller nodes are initialized with their respective rumors in the same way that the initial propagator is initialized. The liar and truth-teller nodes are artificially set to have probability zero of rumor mutation probability zero of rumor acceptance. Thus, only the initial rumor will ever be in their memory banks, and they will only ever spread the initial rumor.

4 Results

The experiments conducted in this work aim to illuminate the effect of consistent actors on rumor spreading, as compared to the metrics in the original work. These metrics are the average entropy over the memory of all nodes, given by

$$\bar{H} = \frac{1}{N} \sum_{n} H_n$$

and opinion fragmentation, i.e. the proportion of nodes that have a particular rumor as the most frequent rumor in memory. With these, the following experiments explore how the consistent actors affect the spread of rumors. In particular, these experiments examine the effects of choice of the proportion of nodes as consistent.

We set $\beta = 1$ to make highly connected nodes more trustworthy. We also set $K = 1$, as this value yielded interesting results in the original work. Finally, we set the number of time steps at $T = 300$. The system was highly susceptible to random fluctuations; having a mutation in the first few time steps almost never lead to convergence on the true rumor, while avoiding an early mutation gave very different results. To combat this, the results shown below are taken as the mean over 100 iterations. Note that the original work did not state the total number of time steps used, and averaged the results over $1,000$ iterations; we selected our values due to computational limitations.

First, we compared the evolution of the average entropy across all nodes, as well as the minimum and maximum entropy across nodes, between the control model and a model with one liar node and one truth-teller node, each chosen uniformly at random. The results are shown in Figure 1, Figure 2, and Figure 3. The presence of the consistent actors has an insignificant impact on the entropy within nodes' memory, but increases the proportion of nodes that believe the truth and the lie. The evolution of the entropy of nodes' memory was insignificantly impacted. The

Evolution of Entropy with One Liar and One Truth-Teller

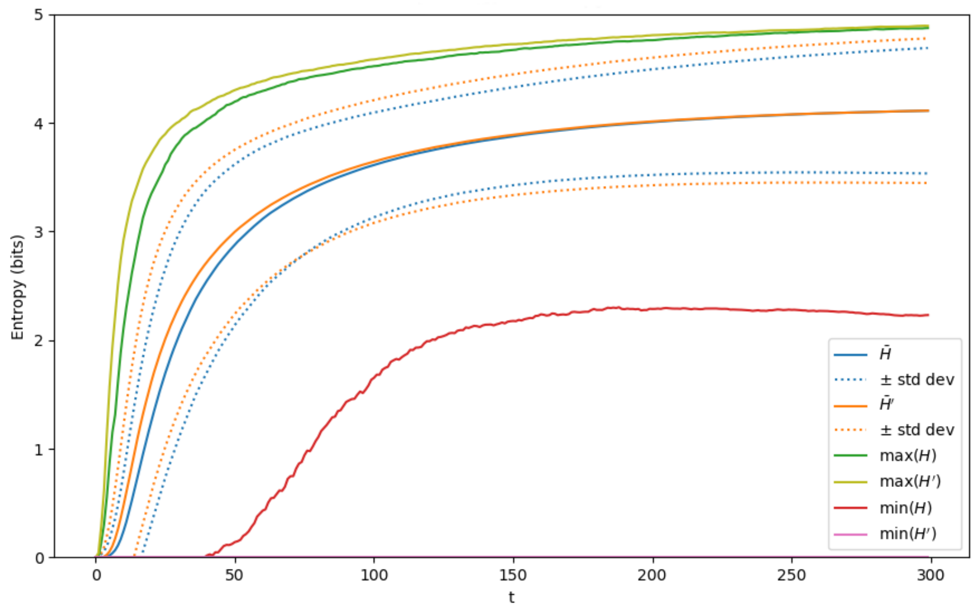

Figure 1: The evolution of entropy in the control model versus in a model with one liar node and one truth-teller node. Note that, while the model with consistent actors is initially higher entropy, the difference becomes insignificant as the model approaches its steady state.

maximum entropy and average entropies are near identical after $t = 100$. The minimum entropies differ, but this is explained by the entropy of a consistent actor necessarily being zero. However, from $t = 100$ to $t = 300$, the proportion of nodes who believed the original rumor in the control experiment fluctuated between 11% and 12%. In contrast, the presence of a truth-telling node and a liar increased this to fluctuate around $18\% \pm 0.5\%$, despite the presence of a liar node.

Increasing the number of consistent actors to fifteen liar nodes and fifteen truth-teller nodes, such that 10% of are consistent actors, has a much more noticeable effect. Opinion fragmentation is dominated by these nodes; approximately 25% and 26% believe the truth and the lie, respectively. The average entropy is initially greater, as initializing a larger number of nodes with rumors leads to faster spreading of rumors; in contrast, in the control model, it takes a greater number of time steps for nodes to hear their first rumor. After an initial spreading period, the average entropy of the model with consistent actors is lower. The value of the entropy in the experimental model at $t = 300$ is 3.76 bits, while the control model entropy is 4.11 bits. However, this can be almost entirely be accounted for by the fact that 10% of the nodes are consistent actors and so have zero entropy; adjusting for this suggests that 270 nodes in the control model would have average entropy of 3.70 bits, nearly identical to the control model. From this, we can conclude that consistent actors have an insignificant impact the entropy of the memory of other nodes, but are able to sway dominant opinions.

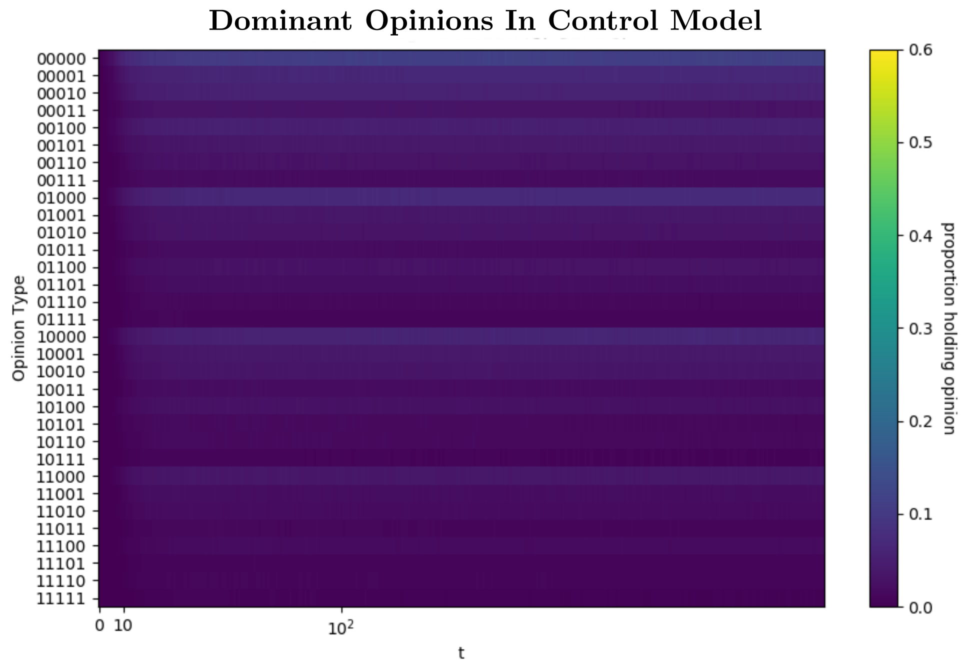

Figure 2: The evolution of dominant opinions in the control model as a function of time. Note that the original rumor $'00000'$ has the greatest proportion of believers at roughly 11%, with rumors with Hamming distance one from this belief comprising the majority of the remaining beliefs.

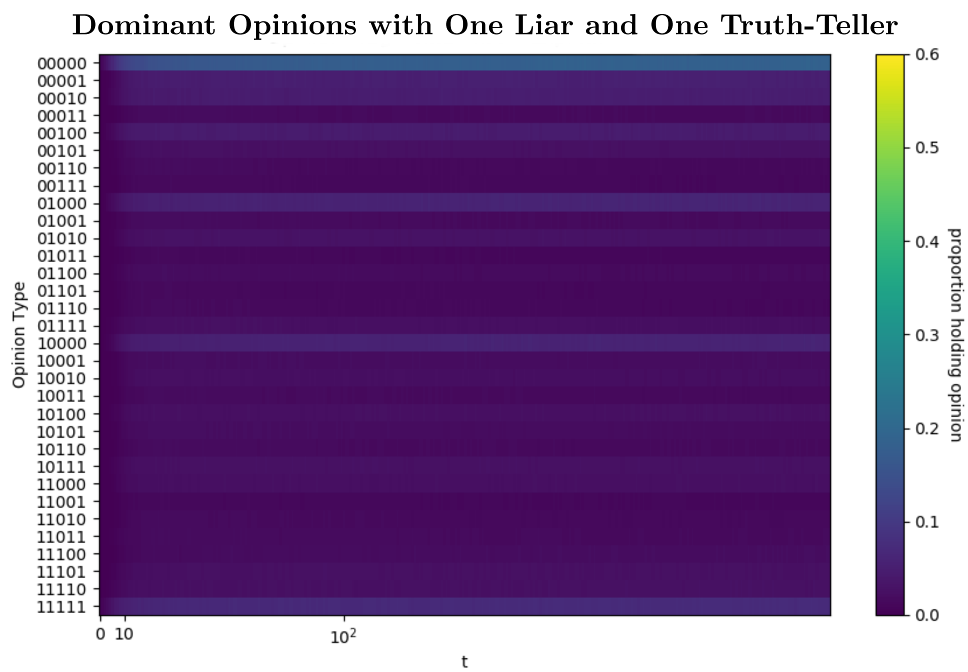

Figure 3: The evolution of dominant opinions as a function of time for a model with one liar node and one truth-teller node. Note that having a truth-telling node increased the proportion of believers in the true rumor $'00000'$ to roughly 18%, despite the presence of the liar node.

Dominant Opinions with Fifteen Liars and Fifteen Truth-Tellers

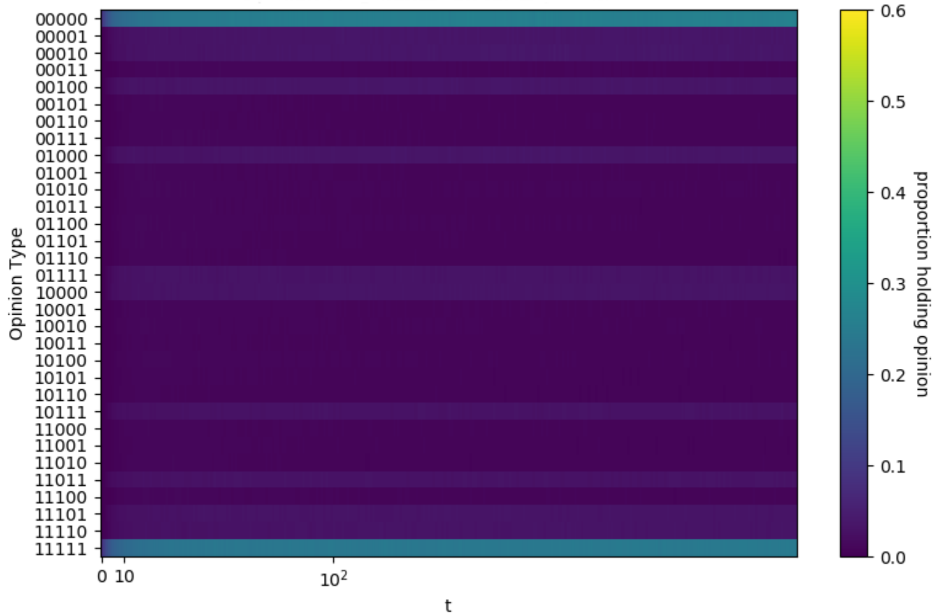

Figure 4: The evolution of dominant opinions as a function of time for a model with fifteen liar nodes and fifteen truth-teller nodes. Note that the prevailing opinions are entirely dominated by the "true" and "false" rumors, at approximately 25% and 26% of the population, respectively.

Evolution of Entropy with Fifteen Liars and Fifteen Truth-Tellers

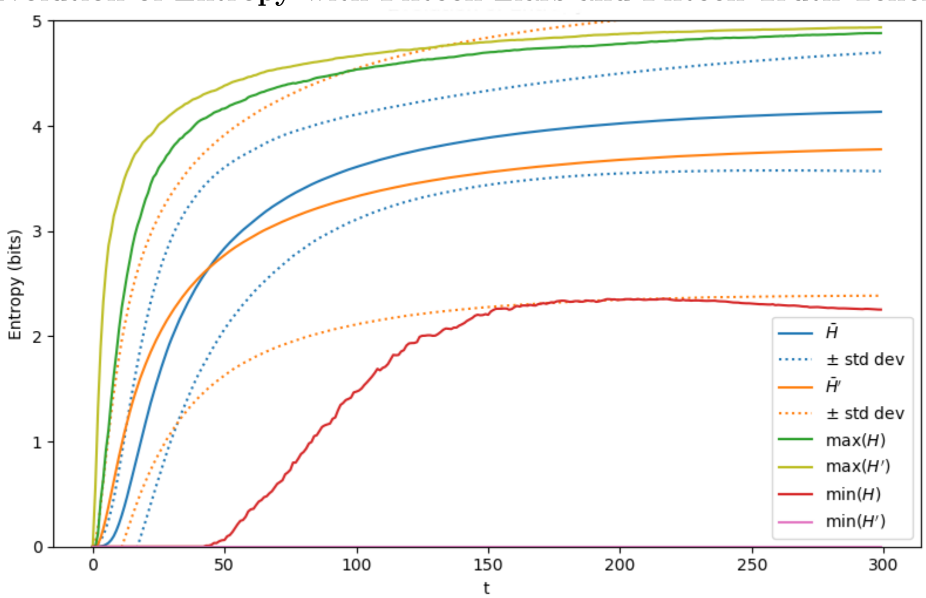

Figure 5: The evolution of entropy in the control model versus in a model with fifteen liar nodes and fifteen truth-teller nodes. Note that the model with consistent actors ultimately achieves a lower entropy, but this lower entropy is nearly identical to the entropy of the control model with the consistent actors removed.

5 Conclusion

Through this work, we see that the impact consistent actors have on a rumor spreading model depends on the ratio of consistent actors to normal nodes. When there are few consistent actors, the average entropy of the model is not significantly impacted, and dominant opinions shift slightly but noticeably towards the rumors spread by the consistent actors. When the number of consistent actors increases, the opinions of other nodes are dominated by the rumors they spread. However, the entropy within the memory of nodes is largely unaffected by the presence of consistent actors. This suggests that the normal nodes are still exposed to the same variety of rumors as in the control model, but being fed the "true" or "false" rumor repeatedly nudges the dominant opinion to conform to those particular rumors.

While this work provides an idea of the effects of consistent actors, there are far more dynamics to be explored. We consider the effects for two particular proportions of consistent actors. Further work could investigate the dynamics with respect to changing this hyperparameter, as the effects may be nonlinear in the number of consistent actors. Future work could also set the number of liar nodes and truth-teller nodes to be unequal, and observe whether opinion fragmentation reaches a steady-state similar to the chosen proportion or whether one rumor comes to dominate. In addition, consistent actors are chosen at random in this work; however, in practical applications, consistent actors may be advertisers given sponsorships because of their widespread influence. Thus, intentionally setting consistent actors as hub nodes would be another direction to take this work. Finally, we have set the consistent actors as either entirely truth-tellers spreading $'00000'$ or entirely liars spreading $'11111'$. Having multiple types of rumor spreaders, or choosing rumors that have less than the maximum possible Hamming distance between them, could yield dominant opinions that lie between the rumors being spread. This work takes the first steps towards these goals by introducing a model for consistent actors.

References

[1] A.-L. Barabási, *Network Science*, ch. 5. Cambridge University Press, 2015.

[2] C. Wang, Z. X. Tan, Y. Ye, L. Wang, K. H. Cheong, and N.-g. Xie, "A rumor spreading model based on information entropy," *Scientific Reports*, vol. 7, pp. 2045–2322, Aug 2017.